Reading Questions 4

1. The comparison test is a test that can be used to determine if a series converges. T
2. A $p-$series converges if $p > 1$ and diverges if $p \leq 1$. T
3. The limit comparison test requires the terms of the series to be positive. T
4. Which test was used in the example?

Section 9.4 Tests for Convergence

9.4 Worksheet

P1. $\sum_{n=1}^{\infty} \frac{20}{n^{20}}$ will converge by the p series test because $P=20$ and $20 > 1$

Comparison Test

P 1. When choosing a test to determine if a series converges or diverges you should look for common patterns in the terms. Does the series $\sum_{n=1}^{\infty} \frac{20}{n^{20}}$ converge or diverge.

P 2. Is the sequence $\frac{n^2-5}{n^3+n+2}$ larger or smaller than the sequence $\frac{1}{n}$? Justify your answer by using inequalities? $n \geq 1$

$\boxed{P2}$ $\left(\frac{n}{n}\right)\frac{n^2-5}{n^3+n+2} \overset{?}{=} \frac{1}{n}\left(\frac{(n^3+n+2)}{(n^3+n+2)}\right)$

$\frac{n(n^2-5)}{n(n^3+n+2)} \overset{?}{=} \frac{n^3+n+2}{n^4+n^2+2n}$

$\frac{n^3-5n}{\cancel{n(n^3+n+2)}} \overset{?}{=} \frac{n^3+n+2}{\cancel{n(n^3+n+2)}}$

$\cancel{n^3}-5n \overset{?}{=} \cancel{n^3}+n+2$

$-5n \overset{?}{=} n+2$

$n \overset{?}{=} \frac{-n}{5} - \frac{2}{5}$

$n > \frac{-n}{5} - \frac{2}{5}$ for $n \geq 1$

$\boxed{\therefore \frac{n^2-5}{n^3+n+2} > \frac{1}{n}}$

P 3. When determining if a series converges or diverges be sure to state the test being used. Does the series $\sum_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converge or diverge.

Limit Comparison

P 4. Use the Limit Comparison Test to determine if the series $\sum_{n=1}^{\infty} \frac{n+5}{n^2+4}$ converges or diverges. In general, you may use any test to determine if a series converges or diverges.

P 5. Use the Limit Comparison Test to determine if the series $\sum_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converges or diverges.

Tests

geometric

monotone and bounded

divergent

$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} a x^n$

a is real

$|x| < 1$

if $\lim a_n \neq 0$ then $\sum a_n$ diverges

$\lim a_n = S$ $\lim b_n = 0$

$\sum a_n + b_n$ $\lim a_n + b_n = \lim a_n + \lim b_n$

$= S + 0 = S \neq 0$

$\therefore \quad \sum a_n + b_n$ diverges

p-series

<u>Thm:</u> $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

<u>Ex:</u> $\sum \frac{1}{n^3}$ converges by the p-series test

Let $0 \leq a_n \leq b_n$. We know $\sum a_n$ and $\underline{\sum b_n}$ are monotone. Also $\sum a_n \leq \sum b_n$. So if $\sum b_n = L$ then $\sum a_n$ is monotone and bounded. $\therefore$ $\sum a_n$ converges.

<u>comparison test</u>

If $0 \leq a_n \leq b_n$ and $\sum b_n$ converges then $\sum a_n$ converges.

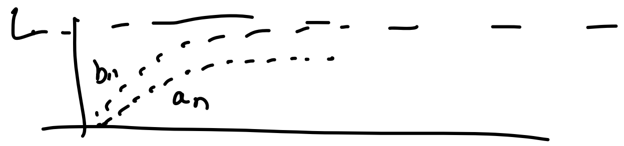
<u>Ex:</u> Let $a_n = \frac{1}{n^3+1}$. Then $b_n = \frac{1}{n^3}$. We know $\sum b_n$ converges by the p-series test. $0 \leq a_n$

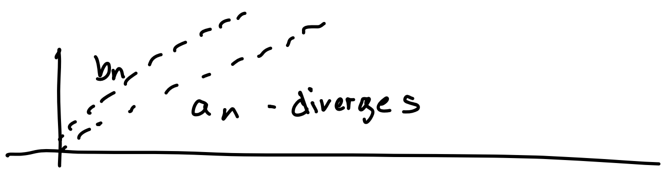
$$n^3 \cdot \frac{1}{n^3+1} \overset{?}{\leq} \frac{1}{n^3} \cdot n^3 \Rightarrow n^3+1 \cdot \frac{n^3}{n^3+1} \leq 1 \cdot n^3+1 \Rightarrow n^3 \leq n^3 + 1$$

$$n^3 \leq n^3 + 1$$

$$0 \leq 1 \checkmark \Rightarrow 0 \leq a_n \leq b_n.$$

$\therefore$ By the comparison test $\sum a_n$ converges.





__Thm:__ $0 \le a_n \le b_n$ If $\sum a_n$ diverges then $\sum b_n$ diverges

__Ex:__ Let $b_n = \dfrac{20}{\sqrt{n}+1}$. Then $0 \le \underset{\underset{a_n}{\parallel}}{\dfrac{20}{\sqrt{n}}} \le b_n$

Note $\dfrac{20}{\sqrt{n}} = \dfrac{20}{n}$. By the p-series test $\sum \dfrac{1}{n}$ diverges since $1 \le 1$. Hence $20 \sum \dfrac{1}{n} = \sum \dfrac{20}{n}$ and $\sum \dfrac{20}{n}$ diverges. Also

$$\frac{20}{n} \overset{?}{\le} \frac{20}{\sqrt{n}+1}$$

$$\Rightarrow \sqrt{n}+1 < n \quad \checkmark$$

$$\Rightarrow (\sqrt{n}+1)^2 < n^2$$

$$\Rightarrow n + 2\sqrt{n} + 1 < n^2$$

By the previous thm $\sum \dfrac{20}{\sqrt{n}+1}$ diverges.