Reading Questions 3

1. If $\sum_{n=1}^{\infty} a_n$ is a series that converges then $\sum_{n=1}^{\infty} ka_n$ converges to $k\sum_{n=1}^{\infty} a_n$. T
2. If $\sum_{n=1}^{\infty} a_n$ is a series that converges then $\sum_{n=2021}^{\infty} a_n$ converges as well. T
3. Were there any explanations since the previous class which were unclear?

Section 9.3 Convergence of Series (Part 1)

Limits of Sequences of Partial Sums

P 1. We will learn many techniques for determining if infinite series converges or diverges. How can partial sums of an infinite series be used to determine if the series converges or diverges?

P 2. Make a list of techniques that determine if a series converges. Which of these techniques can be used to determine if the series diverges?

P 3. Suppose $\lim_{n\to\infty} a_n = 5$ and $\lim_{n\to\infty} b_n = 0$. Does $\sum_{n=1}^{\infty} a_n + b_n$ converge or diverge? Explain your answer.

P 4. Does the series $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$ converge or diverge?

P 5. Does the series $\sum_{n=1}^{\infty} \cos(n)$ converge or diverge?

P 6. Suppose $k \neq 0$. If $\sum_{n=1}^{\infty} a_n$ diverges does $\sum_{n=1}^{\infty} ka_n$ converges or diverge?

P 7. If $\sum_{n=1}^{\infty} a_n$ converges does $\sum_{n=1}^{\infty} a_{n+1} - a_n$ converges or diverge?

Recall

$$S = a + ax + ax^2 + ax^3 + \cdots = \frac{a}{1-x} \qquad |x| < 1$$

$$= \sum_{i=0}^{\infty} ax^i$$

$$S = a(1 + x + x^2 + x^3 + \cdots)$$

$$T = \frac{S}{a} = 1 + x + x^2 + x^3 + \cdots$$

$T_1 = 1 \quad T_2 = 1 + x \quad T_3 = 1 + x + x^2 \quad T_4 = 1 + x + x^2 + x^3$

$T_1, T_2, T_3, T_4, \ldots$

$$T = \lim_{n \to \infty} T_n = \lim_{n \to \infty} 1 + x + x^2 + \cdots + x^{n-1}$$ DNE when $|x| > 1$

$$= \lim_{n \to \infty} \frac{1 \cdot (1 - x^n)}{1 - x}$$ $x \neq 1$

$$= \frac{1}{1-x} \underbrace{\lim_{n \to \infty} 1 - x^n}_{\to 1}$$ if $|x| < 1$

$$= \frac{1}{1-x} = T = \frac{s}{a}$$

$$s = \frac{a}{1-x}$$

Ex:

$1 + 2 + 4 + 8 + \ldots$

$s = 1 + 1 \cdot 2 + 1 \cdot 2^2 + 1 \cdot 2^3 + \ldots$

$a = 1 \quad x = 2$

$s = \frac{1}{1-2} = -1$ This is wrong!! $x > 1$

Ex: $s = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$a = 2 \quad x = \frac{1}{2}$

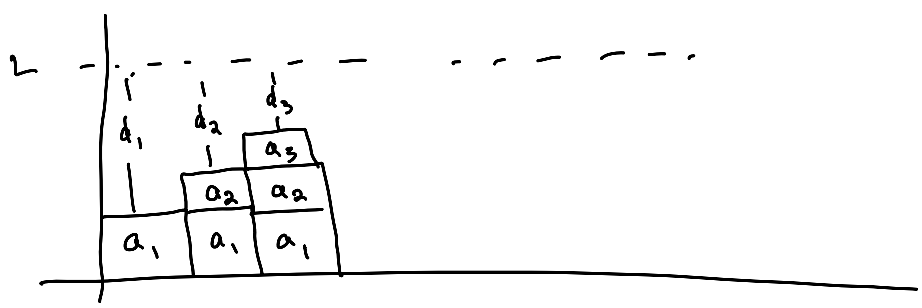
$$s = \frac{2}{1 - \frac{1}{2}} = 4$$

9.3 Convergence of Series

$s = a_1, a_2, a_3, \ldots$ $\qquad S_i = \sum_{j=0}^{i} a_j$

$$\lim_{n \to \infty} S_n = \sum_{i=1}^{\infty} a_i = S$$

Def: If $S_1, S_2, S_3, \ldots$ converges to L then $S = L$. Otherwise S diverges.



$a_n \to 0$ if S exists

Thm:
If $\lim_{n \to \infty} a_n \neq 0$, DNE then $\sum_{n=1}^{\infty} a_n$ diverges.

Thm:
Let k be a real number. Suppose both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge. Then

1) $\sum_{n=1}^{\infty} a_n + b_n$ converges to $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

2) $\sum_{n=1}^{\infty} k a_n$ converges to $k \sum_{n=1}^{\infty} a_n$

Ex: Does $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \frac{1}{4^n}$ converge or diverge.

$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges to $\frac{1/2}{1-\frac{1}{2}}$ since the series $\sum_{n=1}^{\infty} a x^{n-1}$ is geometric

$\sum_{n=1}^{\infty} \left(\frac{1}{4^n}\right) = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ converges to $\frac{1/4}{1-\frac{1}{4}}$

By the previous thm $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \frac{1}{4^n}$ converges

to $\frac{1/2}{1-\frac{1}{2}} + \frac{1/4}{1-\frac{1}{4}} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \frac{1}{4^n}$

Ex: Does $\sum_{n=1}^{\infty} 2^n$ converge or diverge?

By the previous $\lim_{n \to \infty} 2^n$ DNE $\neq 0$

$\therefore \sum_{n=1}^{\infty} 2^n$ diverges