first term $a = 5$
Common ratio $= -2$

The series can be written as: $5 + 5(-2)^1 + 5(-2)^2 + 5(-2)^3 + \dots$

Section 9.2 Geometric Series (Part 1)

Geometric Series

P 1. It is not always obvious that a series is a geometric series. Write the series $5 - 10 + 20 - 40 + 80 + \cdots$ in the form $a + ax + ax^2 + ax^3 + \cdots$.

P 2. Compute the sum of the finite geometric series $2(0.1)^5 + 2(0.1)^6 + \cdots + 2(0.1)^{13}$.

> **Theorem**
>
> The sum of the infinite geometric series
>
> $$S = a + ax + ax^2 + \cdots$$
>
> is $\frac{a}{1-x}$ if $|x| < 1$.

P 3. Compute $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$.

P 4. Find the value of the following geometric sequence.

$$\frac{3}{2} - \frac{1}{2} + \frac{1}{6} - \frac{1}{18} + \frac{1}{54} + \cdots .$$

P 5. If $S = a + ax + ax^2 + ax^3 + \cdots$ is a infinite geometric series which is equal to 8 can $a = 1$?

P 6. We have a way of determining the value of a infinite geometric series when $|x| < 1$. Let $x = 1$. What can be said about $a + ax + ax^2 + ax^3 + \cdots$? What about when $x = -1$?

P 7. Determine the sum of the series

$$S = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots .$$

9.2 Geometric Series

<u>Def:</u> A series is the sum of the terms in a list.

<u>Ex:</u> $s_n = \frac{1}{n}$ for $n \geq 1$ and

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

is a series for s_n.

<u>Ex:</u> $S_4 = 1 + 2 + 4 + 8$ is a series with a finite number of terms.

<u>Def:</u> A finite geometric series is of the form

$$a + ax + ax^2 + ax^3 + \cdots + ax^{n-1} = S_n = \sum_{i=0}^{n-1} ax^i$$

partial sum

where a and x are real numbers.

An infinite geometric series is of the form

$$a + ax + ax^2 + ax^3 + \cdots + ax^{n-1} + \cdots = S = \sum_{i=0}^{\infty} ax^i$$

where a and x are real numbers.

<u>Ex:</u> $S = 1 + 2 + 4 + 8 + 16 + \cdots$ is a geometric series.

$a = 1 \qquad 2 = 1x \Rightarrow x = 2$

$$S = 1 + 1 \cdot 2 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + \cdots$$

$$= \sum_{i=0}^{\infty} 1 \cdot 2^i$$

<u>Ex:</u> $S_4 = 1 + 2 + 4 + 8 = \sum_{i=0}^{3} 1 \cdot 2^i \qquad n = 4 \quad n - 1 = 3$

<u>Thm:</u> $S_n = a + ax + ax^2 + \cdots + ax^{n-1}$

$$= \frac{a(1 - x^n)}{1 - x} \quad \text{if} \quad x \neq 1$$

$$S_n = a + ax + ax^2 + \dots + ax^{n-1}$$

$$= a(1 + x + x^2 + \dots + x^{n-1})$$

$$S_n - xS_n = a(1 + x + x^2 + \dots + x^{n-1}) - a(x + x^2 + x^3 + \dots + x^n)$$

$$\Rightarrow \quad S_n(1-x) = a(1 - x + x - x^2 + x^2 \pm \dots + x^{n-1} - x^{n-1} - x^n)$$

$$= a(1 - x^n)$$

$$\Rightarrow \quad S_n = \frac{a(1-x^n)}{1-x} \qquad \text{where} \quad x \neq 1$$

Ex:

$$S_n = 8 + 4 + 2 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^{16}}$$

$a = 8 \qquad 4 = 8x \Rightarrow \frac{1}{2}$

$$S_n = \frac{8 \cdot (1 - (\frac{1}{2})^{19})}{1 - \frac{1}{2}} \qquad n = 19$$

Thm:

$$S = a + ax + ax^2 + \dots = \frac{a}{1-x} \qquad \text{where} \quad |x| < 1$$

$$S = a(1 + x + x^2 + \dots)$$

$$\frac{S}{a} = 1 + x + x^2 + \dots = T$$

$$T_n = \sum_{i=0}^{n-1} x^i$$

$$T = \lim_{n \to \infty} T_n$$

to be continued.

Ex:

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$a = 1 \qquad \frac{1}{2} = 1 \cdot x \quad \Rightarrow \quad x = \frac{1}{2}$

$$S = \frac{1}{1 - \frac{1}{2}}$$

$T_1 = 1$

$T = \lim T_n$

$T_2 = 1 + \frac{1}{2}$

$T_3 = 1 + \frac{1}{2} + \frac{1}{4}$

$T_4 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$