Office hours MW 3pm - 4pm R 1:30pm - 2:30pm

and by apt.

Reading Questions 2

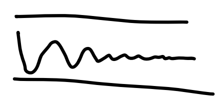
1. If a sequence has a limit then the sequence converges. T
2. As n goes to infinity the sequence $s_n = \frac{1}{n^p}$ converges to 0 for all p. F
3. What is the limit of the function e^{-n} as n goes to infinity? 0 $\frac{1}{e^n}$

Section 9.2 Geometric Series (Part 1)

Monotone Sequences

P 1. One technique for showing that a sequence does not converge is by showing that the sequence is not bounded. State whether the sequence $s_n = \cos(n)$ for $n \geq 1$ is bounded above or below. From your answer can you conclude that the sequence does not converge?

P 2. Determine if the sequences are monotone increasing or decreasing.



1. $s_n = \frac{1}{n^2}$ for $n \geq 1$ $s_{n+1} < s_n$ $\frac{1}{(n+1)^2} < \frac{1}{n^2}$
2. $b_n = \left(\frac{1}{3}\right)^n$ for $n \geq 1$

Geometric Series

P 3. It is not always obvious that a series is a geometric series. Write the series $5 - 10 + 20 - 40 + 80 + \cdots$ in the form $a + ax + ax^2 + ax^3 + \cdots$.

Theorem

The sum of the infinite geometric series

$$S = a + ax + ax^2 + \cdots$$

is $\frac{a}{1-x}$ if $|x| < 1$.

P 4. If $S = a + ax + ax^2 + ax^3 + \cdots$ is a infinite geometric series which is equal to 8 can $a = 1$?

P 5. Determine the sum of the series

$$S = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots .$$

9.1 limits of sequence

Def: Given s_n

a real number

$$\lim_{n\to\infty} s_n = L \quad : \text{the limit of } s_n$$

Ex: Consider $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$$s_n = \frac{1}{n} \qquad n \geq 1$$

$$\lim_{n\to\infty} s_n = 0$$

Def: s_n converges if $\lim\limits_{n\to\infty} s_n = L$ exists.

s_n diverges otherwise.

Ex: Consider $s_n = \frac{1}{n^2}$ for $n \geq 1$. $\frac{1}{n^2}$ behaves like $\frac{1}{n}$

$$\lim_{n\to\infty} \frac{1}{n} = 0 \implies \lim_{n\to\infty} \frac{1}{n^2} = 0$$

Ex: $s_n = n^2$ diverges as $n \to \infty$

Ex: Determine if $s_n = \dfrac{1-e^{-n}}{1+e^{-n}}$ converges or diverges.

$$\lim_{n\to\infty} \frac{1-e^{-n}}{1+e^{-n}} = \frac{\lim 1 - \lim e^{-n}}{\lim 1 + \lim e^{-n}} = \frac{1-0}{1+0} = 1$$

$\therefore$ s_n converges to 1

Def: s_n is bounded below if there exists K such that

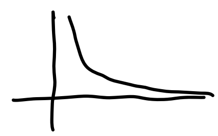
$$K \le s_n \quad \text{for all } n.$$

s_n is bounded above if there exists M such that

$$s_n \le M \quad \text{for all } n.$$

s_n bounded if it is bounded below and above.

Ex: $\frac{1}{n}$ is bounded below by 0. and $-1, -2$



Ex: $s_n = 1 + (-1)^n$ for $n \ge 1$ is bounded.

s_n is bounded below by 0

s_n is bounded above by 2

Def: s_n is monotone increasing if $s_n < s_{n+1}$ for all n.

s_n is monotone decreasing if $s_{n+1} < s_n$ for all n.

s_n is monotone if it is monotone increasing or decreasing

Ex: $s_n = \frac{1}{n}$, $n \ge 1$ $\quad$ s_n is monotone decreasing

<u>Thm:</u> If s_n is bounded and monotone then s_n converges.