Reading Questions 13

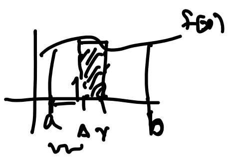
Section 8.1 : Example 1

1. The ratio of the bases of two similar triangles is equal to the ratio of their heights.
2. Right rectangles were used to approximate the isosceles triangle.
3. Why are the bounds on the integral from 0 to 5 instead of 0 to 10?

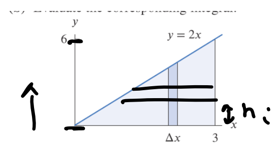
Section 8.1 Areas and Volumes (Part 1)

2D - Areas horizontal slices

P 1. Write a Riemann sum approximating the area of the region in the figure below using vertical strips. Write a Riemann sum approximating the area of the region in the figure below using horizontal strips. Then compute the exact value of the area of the shaded region.

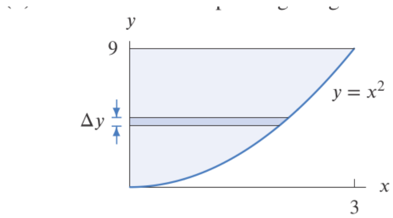


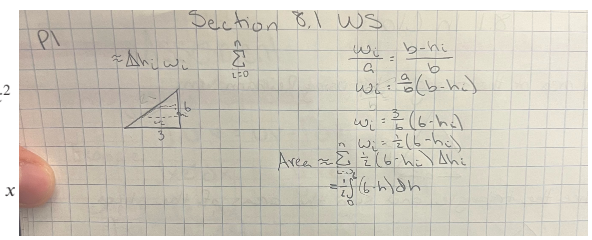
$f(x_i)\Delta x$

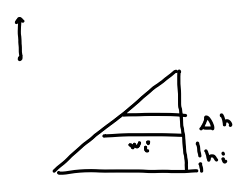


check your work!

P 2. Write a Riemann sum approximating the area of the region in the figure below using horizontal strips. Then compute the exact value of the area of the shaded region.

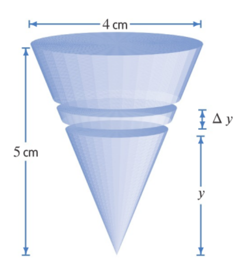




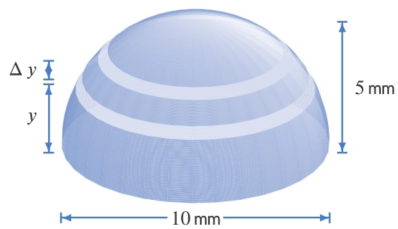


3D - Volumes using Cross Sections

P 3. Use horizontal slicing to find the volume of the figure below.

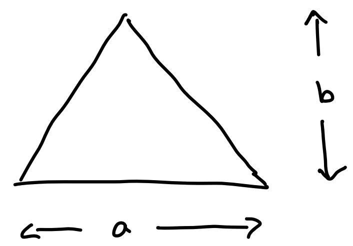


P 4. Use horizontal slicing to find the volume of the figure below.



8.1

Ex: Find the area of the triangle by using Riemann Sums.



slice the triangle and sum the areas of the slices

area of  $\approx \Delta h_i \, w_i$

$$\text{Area of } \Delta \approx \sum_{i=0}^{n} w_i \, \Delta h_i$$

Using similar triangles we get

$$\frac{w_i}{a} = \frac{b-h_i}{b} \Rightarrow w_i = \frac{a}{b}(b-h_i)$$

$$\text{Area of } \Delta \approx \sum_{i=0}^{n} \frac{a}{b}(b-h_i)\,\Delta h_i$$

$$\text{Area of } \Delta = \lim_{n\to\infty} \sum_{i=0}^{n} \frac{a}{b}(b-h_i)\,\Delta h_i$$

$$= \int_0^b \frac{a}{b}(b-h)\,dh \stackrel{?}{=} \frac{ab}{2}$$

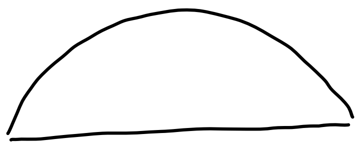
check

$$\frac{a}{b}\int_0^b (b-h)\,dh = \frac{a}{b}\left(bh - \frac{h^2}{2} \Big]_0^b \right)$$

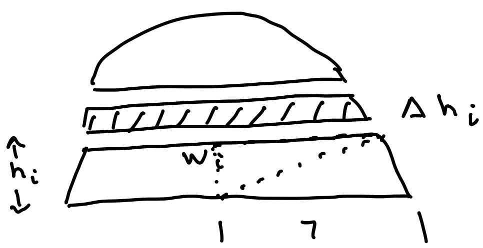
$$= \frac{a}{b}\left(b^2 - \frac{b^2}{2}\right) = ab - ab\cdot\frac{1}{2}$$

$$= \frac{ab}{2} \quad \checkmark$$

Ex: Use horizontal slices to set up a definite integral which represents the area of the semicircle.

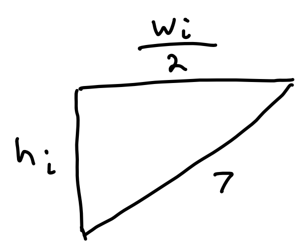


Slice :



Area of ▭ $\approx w_i \Delta h_i$

Write w_i in terms of h_i



$$h_i^2 + \left(\frac{w_i}{2}\right)^2 = 7^2$$

$$w_i = \sqrt{4(7^2 - h_i^2)}$$

$$\Rightarrow \quad w_i = 2\sqrt{49 - h_i^2}$$

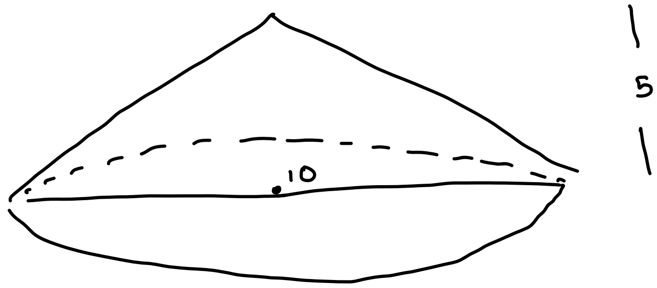
Area of  $\approx \sum_{i=0}^{n} \left(2\sqrt{49 - h_i^2}\right) \Delta h_i$

$\uparrow$
w_i

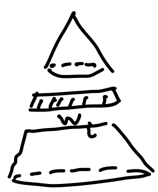
Area of  $= \lim_{n \to \infty} \sum_{i=0}^{n} \left(2\sqrt{49 - h_i^2}\right) \Delta h_i$

$= \int_0^7 2 \cdot \sqrt{49 - h^2}\, dh$

Ex: Using horizontal slices, find the volume of the following cone



slice



Δh_i

volume of  $\Delta h_i \approx \pi\left(\frac{w_i}{2}\right)^2 \Delta h_i$

w_i

Write w_i in terms of h_i

$$\frac{w_i}{10} = \frac{5 - h_i}{5} \Rightarrow w_i = 2(5 - h_i)$$

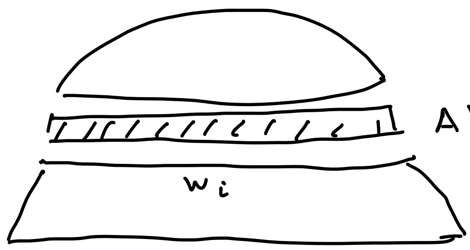
Volume of  $\approx \sum_{i=0}^{n} \pi\left(\frac{2(5-h_i)}{2}\right)^2 \Delta h_i$

Volume of  $= \lim_{n \to \infty} \sum_{i=0}^{n} \pi (5 - h_i)^2 \, \Delta h_i$

$$= \int_0^5 \pi (5-h)^2 \, dh$$

Ex: Find the volume of the following

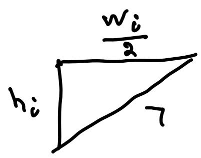
slice



volume of  $\Delta h_i \approx \pi \left(\frac{w_i}{2}\right)^2 \Delta h_i$

w_i

write w_i in terms of h_i



$h_i^2 + \left(\frac{w_i}{2}\right)^2 = 7^2 \implies \left(\frac{w_i}{2}\right)^2 = 7^2 - h_i^2$

$\implies w_i = 2\sqrt{7^2 - h_i^2}$

Volume of  $\approx \sum_{i=0}^{n} \pi \left(\frac{2\sqrt{7^2 - h_i^2}}{2}\right)^2 \Delta h_i$

Volume of  $= \lim_{n \to \infty} \sum_{i=0}^{n} \pi \left(\frac{2\sqrt{7^2 - h_i^2}}{2}\right)^2 \Delta h_i$

$$= \int_0^7 \pi (7^2 - h^2) \, dh$$