Reading Questions 11

Section 7.4 : Example 5

1. Long division was used to compute the integral. T
2. $\frac{x(x+2)+2x}{x+2} = x + \frac{2x}{x+2}$ $\quad \frac{x(x+2)}{x+2} + \frac{2x}{x+2}$ T
3. What is $\int \frac{1}{(1-2x)}\,dx$?

$= -\frac{\ln|2x-1|}{2} + C$

Section 7.4 Algebraic Identities and Trigonometric Substitutions (Part 2)

Worksheet 7.4 Part 2

1) $\int \frac{3}{(x^2+1)(x+2)}\,dx \quad \frac{3}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$3 = (Ax+B)(x+2) + C(x^2+1)$

if $x=-2 \quad 3 = 5C \quad \frac{3}{5} = C$

if $x=0 \quad 3 = 2B + C = 2B + \frac{3}{5} \quad \frac{12}{5} \cdot \frac{1}{2} = B \quad \frac{6}{5} = B$

$Ax^2 + 2Ax + Bx + 2B + Cx^2 + C$

$A + C = 0$ if $C = \frac{3}{5}$ than $A = -\frac{3}{5}$

Long division

P 1. Compute $\int \frac{3}{(x^2+1)(x+2)}\,dx$.

P 2. Consider the integral $\int \frac{x^4+3x^3+2x^2+1}{x^2+3x+2}\,dx$. What is an indication that long division might be of use to solve this integral? Use long division to compute the integral.

$\int \frac{3}{(x^2+1)(x+2)}\,dx = \int \frac{-\frac{3}{5}x}{x^2+1} + \frac{\frac{6}{5}}{x^2+1} + \frac{\frac{3}{5}}{x+2}\,dx$

$= -\frac{3}{10} \ln|x^2+1| + \frac{6}{10x} \ln|x^2+1| + \frac{3}{5} \ln|x+2| + C$

use $\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$

Trigonometric Substitutions

P 3. Use trig substitution to find $\int \frac{1}{\sqrt{9-x^2}}\,dx$.

P 4. Find $\int \sqrt{16-x^2}\,dx$.

P 5. Find $\int \frac{1}{x^2+1}\,dx$ by using trig substitution.

7.4 non-linear factors

Ex: Find $\int \frac{2}{(x^2+1)(x+1)}\, dx.$ $\frac{A}{x^2+1} + \frac{B}{x+1}$

$$\frac{2}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

$$2 = (Ax+B)(x+1) + C(x^2+1)$$

Let $x = -1$

$2 = 0 + 2C \Rightarrow C = 1$

Let $x = 0$

$2 = B + C \Rightarrow 2 = B+1 \Rightarrow B = 1$

Let $x = 1$

$2 = (A+B)2 + C(2) \Rightarrow 2 = (A+1)2 + 1 \cdot 2$

$\Rightarrow 2 = 2A + 2 + 2$

$\Rightarrow 2 = 2A + 4 \Rightarrow A = -1$

$$\int \frac{2}{(x^2+1)(x+1)}\, dx = \int \frac{-x+1}{x^2+1} + \frac{1}{x+1}\, dx$$

$$= \int \frac{-x+1}{x^2+1}\, dx + \int \frac{1}{x+1}\, dx$$

$$= \int \frac{-x}{x^2+1}\, dx + \int \frac{1}{x^2+1}\, dx + \int \frac{1}{x+1}\, dx$$

let $u = x^2+1$ let $u = x+1$

$du = 2x\,dx$ $du = dx$

$\frac{1}{2}du = x\,dx$

$$= \int \frac{-\frac{1}{2}du}{u} + \int \frac{1}{x^2+1}\,dx + \int \frac{du}{u}$$

$$= -\frac{1}{2}\ln|u| + \tan^{-1}x + \ln|u| + c_1$$

$$= -\frac{1}{2}\ln|x^2+1| + \tan^{-1}x + \ln|x+1| + c_2$$

$\frac{5}{3} = 1.06\ldots$

$= 1 + \frac{2}{3}$

$$3\overline{)\,5\,}\quad 1\ r\ 2$$
$$-3$$
$$2$$

Ex: Find $\displaystyle\int \frac{x^3-7x^2+10x+1}{x^2-7x+10}\,dx$

$$x^2-7x+10 \overline{)\, x^3 - 7x^2 + 10x + 1 \,}\quad x$$
$$-x^3 + 7x^2 - 10x$$
$$0 + 1$$

$$\frac{x^3-7x^2+10x+1}{x^2-7x+10} = x + \frac{1}{x^2-7x+10}$$ check

$$\int x\,dx + \int \frac{1}{x^2-7x+10}\,dx$$

$$\frac{x^2}{2} + c_1 + \int \frac{1}{(x-2)(x-5)}\,dx$$

$$\frac{1}{(x-2)(x-5)} = \frac{A}{x-2} + \frac{B}{x-5}$$

$$1 = A(x-5) + B(x-2)$$

Let $x = 5$

$1 = 3B \Rightarrow B = \frac{1}{3}$

Let $x = 2$

$1 = -3A \Rightarrow A = -\frac{1}{3}$

$$\int \frac{1}{(x-5)(x-2)}\,dx = \int \frac{-1/3}{x-2}\,dx + \int \frac{1/3}{x-5}\,dx$$

$$= -\frac{1}{3}\left(\ln|x-2| - \ln|x-5|\right) + c$$

$$= \frac{x^2}{2} - \frac{1}{3}\left(\ln|x-2| - \ln|x-5|\right) + c$$

Ex: Find $\int \frac{1}{\sqrt{1-x^2}}\,dx$.

Let

$x = \sin y \Rightarrow dx = \cos y\,dy$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-\sin^2 y}} \cdot \cos y\,dy$$

Recall

$1 = \sin^2 y + \cos^2 y \Rightarrow 1 - \sin^2 y = \cos^2 y$

$$= \int \frac{1}{\sqrt{\cos^2 y}} \cdot \cos y\,dy$$

$$= \int \frac{1}{\cos y} \cdot \cos y\,dy = \int dy$$

$$= y + c_1$$

$x = \sin y \Rightarrow \sin^{-1} x = y$

$$= \sin^{-1} x + c_2$$

Ex: Find $\int \sqrt{9-4x^2}\, dx$

$$x = \frac{3}{2}\sin y \Rightarrow dx = \frac{3}{2}\cos y\, dy$$

$$\int \sqrt{9-4x^2}\, dx = \int \sqrt{9 - 4\left(\frac{3}{2}\sin y\right)^2} \cdot \frac{3}{2}\cos y\, dy$$

$$= \frac{3}{2}\int \sqrt{9 - 9\sin^2 y} \cdot \cos y\, dy$$

$$= \frac{3}{2}\int \sqrt{9\cos^2 y} \cdot \cos y\, dy$$

$$= \frac{3 \cdot 3}{2}\int \cos y \cdot \cos y\, dy$$

$$= \frac{9}{2}\left(\frac{1}{2}\cos y \sin y + y + c_1\right)$$

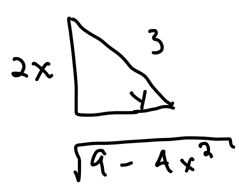
don't do this !!!

$x = \sin y \Rightarrow \cos x = \cos \sin y$

we know $x = \frac{3}{2}\sin y \Rightarrow \sin^{-1}\frac{2x}{3} = y$ We need $\cos y$

$\Rightarrow \frac{2x}{3} = \sin y$

implies the right triangle



$$\left(\sqrt{9-4x^2}\right)^2 + (2x)^2 = 3^2$$

implies $\cos y = \frac{\sqrt{9-4x^2}}{3}$

$$= \frac{9}{2}\left(\frac{1}{2} \cdot \frac{\sqrt{9-4x^2}}{3} \cdot \frac{2x}{3} + \sin^{-1}\frac{2x}{3}\right) + c$$