TA grades for 6
calendar
Khanh

Reading Questions 9

Section 7.2 : Example 1

1. The derivative of $f(x)g(x)$ is $f'(x)g(x) + f(x)g'(x)$. T
2. Suppose I tell you that $\int xe^x\,dx = xe^x - e^x + C$. How can you verify this claim? Find $\frac{d}{dx}\left[xe^x - e^x + c\right]$

Section 7.2 Integration by Parts (Part 1)

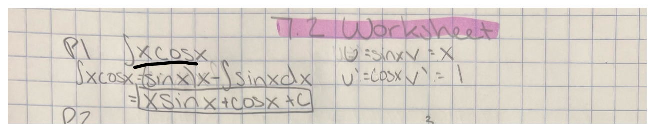
Introduction

Theorem: Integration by parts

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

or

$$\int u'v = uv - \int uv'$$



P 1. Use integration by parts to find $\int x\cos(x)\,dx$.

P 2. Find $\int x^2\ln(x)\,dx$.

P 3. Use integration by parts to find $\int x^2e^{3x}\,dx$. Be sure to write down u and v.

Going in circles

P 4. Find $\int \sin^2(x)\,dx$. Hint: You might find yourself going in circles.

P 5. Find $\int e^x\sin(x)\,dx$.

Let $f'(x) = x^2$ $f(x) = \frac{x^3}{3}$
$g(x) = \ln(x)$ $g'(x) = \frac{1}{x}$

$\frac{x^3}{3} \cdot \ln(x) - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$

$\frac{x^3\ln x}{3} - \frac{1}{3}\int x^2\,dx$

7.2

$$\frac{d}{dx}\left[f(x)\, g(x) \right] = f'(x)\, g(x) + f(x)\, g'(x)$$

$$\int f(x)\, dx = F(x) + C \qquad \text{where} \qquad F'(x) = f(x)$$

Thm: Intergration by part

$$\int f'(x)\, g(x) + f(x)\, g'(x)\ dx = f(x)\, g(x) + c$$

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx$$

$$\int u' v = uv - \int u v'$$

Ex: Find $\int x e^x\, dx$.

Let

$f'(x) = e^x$

$g(x) = x$

because $g'(x)$ is easier to ~~work with~~ integrate.

$f(x) = e^x \qquad g'(x) = 1$

$$\int x e^x\, dx = e^x \cdot x - \int e^x \cdot 1\, dx$$

$$= x e^x - \int e^x\, dx = x e^x - e^x + c$$

check

$$\frac{d}{dx}\left[x e^x - e^x + c \right] = 1 \cdot e^x + x \cdot e^x - e^x + 0$$

$$= x \cdot e^x$$

✓

Ex: Find $\int_2^3 \ln(x)\,dx$.

First find $\int \ln(x)\,dx$

Let

$f'(x) = 1$

$g(x) = \ln(x)$

so

$f(x) = x$

$g'(x) = \frac{1}{x}$

Then

$$\int 1 \cdot \ln(x)\,dx = x\ln(x) - \int x \cdot \frac{1}{x}\,dx$$

$$= x\ln(x) - \int dx$$

$$= x\ln(x) - x + C$$

$$\int_2^3 \ln(x)\,dx = x\ln(x) - x \Big]_2^3 = 3\ln 3 - 3 - 2\ln 2 + 2$$

$$=$$

Ex: Find $\int \cos^2(x)\,dx$.

Let

$f'(x) = \cos x$ $\Rightarrow$ $f(x) = \sin x$

$g(x) = \cos x$ $\qquad$ $g'(x) = -\sin x$

IBP

$$\int \cos^2(x)\,dx = \sin x \cdot \cos x - \int \sin x \cdot (-\sin x)\,dx$$

$$= \sin x \cos x + \int \sin^2 x\,dx$$

Let

$f'(x) = \sin x$ $\Rightarrow$ $f(x) = -\cos x$

$g(x) = \sin x$ $\qquad$ $g'(x) = \cos x$

IBP

$$\int \sin^2 x\,dx = -\cos x \cdot \sin x - \int (-\cos x) \cdot \cos x\,dx$$

$$= -\cos x \sin x + \int \cos^2 x\,dx$$

$$\int \cos^2 x\,dx = \sin x \cos x + \sin x \cos x - \int \cos^2 x\,dx$$

$$+ \int \cos^2 x\,dx \qquad\qquad + \int \cos^2 x\,dx$$

$$2\int \cos^2 x\,dx = 2\sin x \cos x + c_1$$

why is this wrong?

$$\int \cos^2 x\,dx = \sin x \cos x + c_2$$

$$= \sin x \cos x + \int \sin^2 x\,dx$$

since

$$= \sin x \cos x + \int (1 - \cos^2 x)\,dx \qquad 1 = \sin^2 x + \cos^2 x$$

$$\int \cos^2 x\,dx = \sin x \cos x + x - \int \cos^2 x\,dx$$

$$+ \int \cos^2 x\,dx \qquad\qquad + \int \cos^2 x\,dx$$

$$2\int \cos^2 x\,dx = \sin x \cos x + x + c_1$$

$$\int \cos^2 x\,dx = \frac{1}{2}\sin x \cos x + \frac{1}{2}x + c_2 .$$