Reading Questions 8

Section 7.1 : Example 1

1. The derivative of $\cos(x)$ is $\sin(x)$. F $-\sin(x)$
2. The derivative of $f(g(x))$ is $f'(x)g(x) + f(x)g'(x)$. F

$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

3. Compute $\int \cos(x)\, dx$. $= \sin(x) + c$

Section 7.1 Integration by Substitution (Part 1)

Indefinite Integrals

P 1. Which kind of functions would integration by substitution be useful to use to integrate the function?

$f(g(x))$

Theorem

$$\int f'(g(x)) \cdot g'(x)\, dx = f(g(x)) + C$$

P 2. Find $\int 2x \sin(x^2+1)\, dx$. Be sure to clearly state your w and dw.

P 3. Find $\int (3x^2+1)\sqrt{x+x^3}\, dx$.

P 4. Find $\int \frac{1+e^t}{t+e^t}\, dt$.

2) $\int 2x \sin(x^2+1)\, dx$ $w = x^2+1$ $dw = 2x\, dx$

$\int \sin(w)\, dw = -\cos(x^2+1) + C$

3) $\int (3x^2+1)\sqrt{x+x^3}\, dx$ $w = x+x^3$ $dw = 3x^2+1\, dx$

$\int w^{1/2}\, dw = \frac{2}{3}(x+x^3)^{3/2} + C$

Definite Integrals

P 5. Compute $\int_0^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\, dt$ definite integral. Try using both methods from the lecture.

P 6. Find $\int_0^2 \frac{x}{(1+x^2)^2}\, dx$.

P 7. Find $\int x\sqrt{x+1}\, dx$.

P 8. Find $\int (x+7)\sqrt[3]{3-2x}\, dx$.

P2. $\int 2x \sin(x^2+1)\, dx$

$w = x^2+1$

$dw = 2x\, dx$

$\int 2x \sin(x^2+1)\, dx = \int \sin(w)\, dw$

Integrating: $\int \sin(w)\, dw = -\cos(w) + C$

Substituting $w = x^2+1$

$\int \sin(w)\, dw = -\cos(x^2+1) + C$

7.1

<u>Recall:</u> $\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

$$\frac{d}{dx}\left[\underbrace{h(k(x))}_{F(x)}\right] = \underbrace{h'(k(x)) \cdot k'(x)}_{f(x)}$$

<u>Thm:</u>

$$\int h'(k(x)) \cdot k'(x)\,dx = h(k(x)) + C.$$

<u>Ex:</u> Find $\int 3x^2 \cos(x^3)\,dx$.

$w = k(x) = x^3$ $\quad$ $h'(x) = \cos(x)$ $\quad$ $w = x^3$

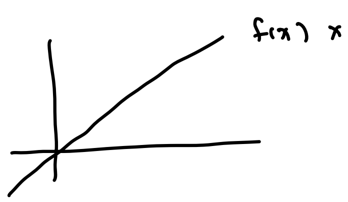
$k'(x) = 3x^2$ $\quad$ $h(x) = \sin(x)$ $\quad$ $\frac{dw}{dx} = 3x^2 \Rightarrow dw = 3x^2\,dx$

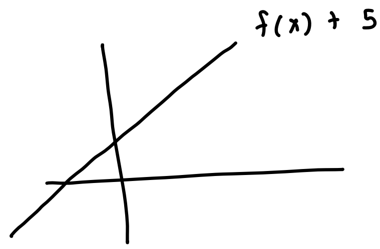
$$\int \underbrace{\cos(x^3)}_{w} \cdot \underbrace{3x^2\,dx}_{dw} = \int \cos w \; dw$$

$$= \sin w + c_1$$

$$= \sin x^3 + c_2$$

check by computing $\frac{d}{dx}\left[\sin x^3 + c_2\right]$





Ex: Find $\int x^3 \sqrt{x^4+5}\, dx$.

$w = x^4 + 5$

$dw = 4x^3\, dx$

$\frac{1}{4} dw = x^3\, dx$

$$\int \sqrt{x^4+5} \cdot x^3 dx = \int \sqrt{w} \cdot \frac{1}{4} dw$$

$$= \frac{1}{4}\int w^{1/2}\, dw$$

$$= \frac{1}{4}\left(\frac{w^{3/2}}{3/2} + c_1\right)$$

$$= \frac{w^{3/2}}{6} + c_2$$

$$= \frac{(x^4+5)^{3/2}}{6} + c_3$$

$\int e^{\cos\theta} \sin\theta\, d\theta$

$w = \cos\theta$

$\int \frac{e^x}{1+e^x}\, dx$

$w = 1+e^x$

$\int \tan\theta\, d\theta = \int \frac{\sin\theta}{\cos\theta}\, d\theta$

$w = \cos\theta$

Ex: Compute $\int_0^2 x e^{x^2}\, dx$

Method 1

By FTC $\int_0^2 x e^{x^2} dx = F(2) - F(0)$

$\int x e^{x^2} dx$

$w = x^2 \quad dw = 2x\, dx$

$\frac{1}{2} dw = x\, dx$

$\Rightarrow$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^w\, dw$$

$$= \frac{1}{2}(e^w + c_1)$$

$$= \frac{1}{2} e^{x^2} + c_2$$

$$\int_0^2 x e^{x^2}\,dx = \frac{1}{2}e^{(2)^2} - \frac{1}{2}e^{0^2} = \frac{1}{2}(e^4 - 1)$$

Method 2

$$\int_0^2 x e^{x^2}\,dw$$

$w = x^2 \quad \frac{1}{2}dw = x\,dx$

$w(0) = 0^2 = 0 \qquad w(2) = 4$

$$= \frac{1}{2}\int_0^4 e^w\,dw = \frac{1}{2}\left(e^w\Big]_0^4\right)$$

$$= \frac{1}{2}(e^4 - e^0)$$

Ex: Find $\int \sqrt{1+\sqrt{x}}\,dx$

$w = 1 + \sqrt{x} \qquad dw = \frac{1}{2}x^{-1/2}\,dx$ ← not in integrand

$w - 1 = \sqrt{x} \implies (w-1)^2 = x$

$\implies 2(w-1)\,dw = dx$

$$\int \sqrt{1+\underbrace{\sqrt{x}}_{w}}\,\underbrace{dx}_{2\cdot(w-1)\,dw} = \int \sqrt{1+\sqrt{(w-1)^2}}\cdot 2(w-1)\,dw$$

$$= 2\int \sqrt{1 + w - 1}\cdot(w-1)\,dw$$

$$= 2\int \sqrt{w}\cdot(w-1)\,dw$$

$$= 2\int w^{3/2} - w^{1/2}\,dw$$

$$= 2\left(\frac{w^{5/2}}{5/2} - \frac{w^{3/2}}{3/2} + c_1\right)$$

$$= 4\left(\frac{(1+\sqrt{x})^{5/2}}{5} - \frac{(1+\sqrt{x})^{3/2}}{3} + c_2\right)$$