Section 9.1 Sequences (Part 1)

P 1. The nth term of a sequence can be used to name the sequence without listing the terms of the sequence. What is the nth term of the sequence $1, 2, 4, 8, 16, \ldots$?

P 2. To determine the nth term of a sequence it is useful to know the first few terms of the sequence in order to find common sequence patterns. For the following sequence write the first six terms and the nth term of the sequence.

$$s_n = ns_{n-1} \quad \text{where} \quad n \geq 2 \text{ and } s_1 = 1.$$

P 3. Determine if the sequences converge or diverge. If the sequences converge find the limit. If the sequence diverges state how you came to that conclusion.

1. $s_n = \frac{n}{n^2+1}$
2. $a_n = \frac{n^3+2}{n^2+1}$
3. $b_n = n + \frac{1}{n}$
4. $c_n = (-1)^n + \frac{1}{n}$

P 4. Give a sequence that diverges which contains positive and negative terms. If the signs of the terms alternate then the sequence is called an alternating sequence.

Section 9.1 Sequences (Part 2)

P 5. One technique for showing that a sequence does not converge is by showing that the sequence is not bounded. State whether the sequence $s_n = \cos(n)$ for $n \geq 1$ is bounded above or below. From your answer can you conclude that the sequence does not converge?

P 6. Determine if the sequences are monotone increasing or decreasing.

1. $s_n = \frac{1}{n^2}$ for $n \geq 1$
2. $b_n = (\frac{1}{3})^n$ for $n \geq 1$

Section 9.2 Geometric Series (Part 1)

P 7. Compute the sum of the finite geometric series $2(0.1)^5 + 2(0.1)^6 + \cdots + 2(0.1)^{13}$.

P 8. Compute $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \cdots$.

P 9. Find the value of the following geometric sequence.

$$\frac{3}{2} - \frac{1}{2} + \frac{1}{6} - \frac{1}{18} + \frac{1}{54} + \cdots.$$

P 10. Determine the sum of the series

$$S = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots.$$

Section 9.3 Convergence of Series (Part 1)

P 11. Suppose $\lim\limits_{n\to\infty} a_n = 5$ and $\lim\limits_{n\to\infty} b_n = 0$. Does $\sum\limits_{n=1}^{\infty} a_n + b_n$ converge or diverge? Explain your answer.

P 12. Does the series $\sum\limits_{n=1}^{\infty} \frac{n+1}{2n+3}$ converge or diverge?

P 13. Does the series $\sum\limits_{n=1}^{\infty} \cos(n)$ converge or diverge?

Section 9.4 Tests for Convergence (Part 1)

P 14. When choosing a test to determine if a series converges or diverges you should look for common patterns in the terms. Does the series $\sum\limits_{n=1}^{\infty} \frac{20}{n^{20}}$ converge or diverge.

P 15. When determining if a series converges or diverges be sure to state the test being used. Does the series $\sum\limits_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converge or diverge.

P 16. Use the Limit Comparison Test to determine if the series $\sum\limits_{n=1}^{\infty} \frac{n+5}{n^2+4}$ converges or diverges. In general, you may use any test to determine if a series converges or diverges.

P 17. Use the Limit Comparison Test to determine if the series $\sum\limits_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converges or diverges.

Section 9.4 Tests for Convergence (Part 2)

P 18. Use the Limit Comparison Test to determine if the series $\sum\limits_{n=1}^{\infty} \frac{n+5}{n^2+4}$ converges or diverges. In general, you may use any test to determine if a series converges or diverges.

P 19. Use the Limit Comparison Test to determine if the series $\sum\limits_{n=1}^{\infty} \frac{n+2}{n^4+n+1}$ converges or diverges.

P 20. Use the absolute value test to determine if the series $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+1}$ converges or diverges.

P 21. Determine if the series $1 - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \cdots$ converges or diverges.

P 22. Is the statement "If $\sum\limits_{n=1}^{\infty} a_n$ converges then $\sum\limits_{n=1}^{\infty} |a_n|$ converges" always true. Explain your answer.

Section 9.4 Tests for Convergence (Part 3)

P 23. Up until this section, the sequence a_n of a series has not contained a factorial. Hence if the sequence contains a factorial that might be an indication to use the Ratio Test.

Determine if the series $\sum\limits_{n=1}^{\infty} \frac{2}{(n+4)!}$ converges or diverges.

P 24. Can the Ratio Test be used to determine if the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$ converges or diverges?

P 25. Determine if the series $\sum_{n=1}^{\infty}\frac{6}{n+2^n}$ converges or diverges. Don't forget to first determine if $\lim_{n\to\infty} a_n = 0$.

P 26. Determine if the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$ converges or diverges. Be sure to state any test that you use.

P 27. Can the alternating series test be used to determine if the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n!}{n^3+2}$ converges or diverges? If so, state whether the series converges or diverges.

Section 9.4 Tests for Convergence (Part 4)

P 28. Estimate the error in approximating the sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}(\frac{1}{3})^{n-1}$ by the sum of the first 4 terms. Check your answer.

P 29. Let $S = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}$. Give a number k such that $|S - \frac{31}{36}| < k$.

P 30. Determine if the series $\sum_{n=1}^{\infty}(-1)^{n-1}(\frac{2}{3})^n$ is absolutely or conditionally convergent.

P 31. Determine if the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$ is absolutely or conditionally convergent.

Section 9.5 Power Series and Interval of Convergence (Part 1)

P 32. Use $\sum$ notation to write the series

$$\frac{1}{2}x + \frac{1}{2^2 \cdot 2!}x^2 + \frac{1}{2^3 \cdot 3!}x^3 + \cdots .$$

State C_n and a.

P 33. Find the interval of convergence for the power series $\sum_{n=0}^{\infty}(\frac{x+1}{2})^n$.

P 34. Determine the radius of convergence of the series

$$\frac{(x-1)}{2} - \frac{(x-1)^2}{2 \cdot 2^2} + \frac{(x-1)^3}{3 \cdot 2^3} - \frac{(x-1)^4}{4 \cdot 2^4} + \cdots + (-1)^n\frac{(x-1)^n}{n \cdot 2^n} + \cdots .$$

State a and C_n and a_n.

P 35. When the series is not geometric, to determine the interval of convergence you must determine if the series converges at the endpoints. Determine the radius and interval of convergence of the series $\sum_{n=0}^{\infty} n^3 x^n$.

P 36. Determine the interval of convergence of the series $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!}$.